Giornale Italiano di Educazione alla Salute, Sport e Didattica Inclusiva / Italian Journal of Health Education, Sports and Inclusive Didactics - Anno 6 n. 4 - ISSN 2532-3296 ISBN 978-88-6022-456-9 - ottobre - dicembre 2022 - CC BY-NC-ND 3.0 IT- https://doi.org/10.32043/gsd.v6i4.706

LEARNING MATHEMATICAL PROOF IN HIGHER EDUCATION: A QUESTION OF POLYSEMY? AN EXPLORATORY STUDY IN FRENCH WEST INDIES

APPRENDERE LA DIMOSTRAZIONE MATEMATICA NELL'ISTRUZIONE SUPERIORE: UNA QUESTIONE DI POLISEMIA? UNA RICERCA ESPLORATIVA NELLE ANTILLE FRANCESI

Mickaelle Ramassamy
Université des Antilles
mickaelle.ramassamy@univ-antilles.fr

Antoine Delcroix
Université des Antilles
antoine.delcroix@univ-antilles.fr

Abstract

The notion of proof is a pivotal issue in mathematical teaching. In France, school curricula have been associating this notion with the term *Demonstration*. The term *Preuve* is lesser used. In French, the words *démontrer*, *justifier*, *prouver* are considered equivalents, but some researchers emphasize differences between them. For indeed, Balacheff and Soury-Lavergne (1996) consider that a proof is an accepted explanation, whereas the word *Démonstration* designates an explanation accepted by mathematicians because of its particular structure.This polysemy could have an impact on mathematical learning, especially in ultramarine contexts, where learners are exposed to multicultural and multilingual environments but are asked to follow the same curricula as in mainland. Following our hypothesis, we conduct a longitudinal survey in a French ultramarine context (the French West Indies): 168 students are questioned about their conception of the meaning of the terms *Demonstration* and *Justification* at three steps of their course. They were also questioned about the evolution of their ability to argue, reason, and write a proof. We observed the evolution of their abilities. We also established causal links between the various observed conceptions of students on the meaning of these terms and the evolution of their capacity. The results we obtained suggest that there is some adequacy between these conceptions and perception of how their ability to demonstrate a result in mathematics evolve.

La nozione di dimostrazione è una questione centrale nell'insegnamento della matematica. In Francia, i programmi scolastici hanno associato questa nozione al termine Dimostrazione. Il termine Preuve è meno usato. In francese, le parole démontrer, justifier, prouver sono considerate equivalenti, ma alcuni ricercatori sottolineano le differenze tra di loro. Infatti, Balacheff e Soury-Lavergne (1996) ritengono che una dimostrazione sia una spiegazione accettata, mentre la parola Démonstration designa una spiegazione accettata dai matematici a causa della sua particolare struttura.Questa polisemia potrebbe avere un impatto sull'apprendimento della matematica, specialmente nei contesti oltremare, dove gli studenti sono esposti ad ambienti multiculturali e multilingue ma sono invitati a seguire gli stessi curricula della terraferma. Seguendo la nostra ipotesi, conduciamo un'indagine longitudinale in un contesto oltremare francese (le Indie occidentali francesi): 168 studenti vengono interrogati sulla loro concezione del significato dei termini Dimostrazione e Giustificazione in tre fasi del loro corso. Sono stati anche interrogati sull'evoluzione della loro capacità di argomentare, ragionare e scrivere una dimostrazione. Abbiamo osservato l'evoluzione delle loro capacità. Abbiamo anche stabilito nessi causali tra le diverse concezioni osservate dagli studenti sul significato di questi termini e l'evoluzione delle loro capacità. I risultati che abbiamo ottenuto suggeriscono che esiste una certa adeguatezza tra queste concezioni e la percezione di come si evolve la loro capacità di dimostrare un risultato in matematica.

Giornale Italiano di Educazione alla Salute, Sport e Didattica Inclusiva / Italian Journal of Health Education, Sports and Inclusive Didactics - Anno 6 n. 4 - ISSN 2532-3296 ISBN 978-88-6022-456-9 - ottobre - dicembre 2022 - CC BY-NC-ND 3.0 IT- https://doi.org/10.32043/gsd.v6i4.706

Key-words

Didactics of mathematics, higher education, mathematical proof, Overseas France

Didattica della matematica, Francia d'oltremare, istruzione superior, prova matematica

Introduction

According to an anthropological approach, mathematical objects must be transformed by the didactic transposition process (Chevallard, 1985) to be taught. This process leads to build the teaching text about the object. The understanding of this text, and specific terms around it, is necessary for better learning of the object. In the French language, different polysemic words are used in teaching practices leading to learning of mathematical proof. More precisely, different words are used to talk about proof such as *Démontrer* or *Prouver*. In the English context, the word "prove" is mainly used. In the context of Canadian education, the term proof is less used, but the teaching of reasoning in mathematics will be more mentioned. Moreover, in some French schoolbooks for secondary level, exercises ask to "justify", "prove", "show that" or also use the French word *Démontrer* (but neither curricula nor books describe the difference between these terms). The study of Baron and Hache (2019) highlights various meanings that teachers give to each word used in these questions.

Because of these different meanings of the terms *Démontrer* and *Justifier*, students may understand an exercise differently when teachers ask to justify or proof. In secondary school, teachers' different conceptions of justification or proof tend to build students' conceptions. Students begin their course at the higher education level with these conceptions about concepts around proof. Moreover, the difference between teaching in secondary and higher education may lead to a gap. The proof which is taught in secondary schools is different to expert practices of mathematicians, which influence mathematical teaching practices in higher education.

Due to differences in expectations with respect to the proof and in pedagogical practices, it is interesting to observe how students evolve when they begin their course. Studying their conceptions of the terms involved in learning the proof at the beginning of the course is a first step to study this evolution. A second step leads us to observe difficulties perceived by students when they must prove a result.

More specifically, the goal of this research is to study the implications of polysemy of terms on the learning of proof in higher education. In the first part, we describe the theoretical frame and expose our hypotheses. Then, we describe the methodology. In the third and fourth parts, we describe and discuss the results. In the last part, we point out some limitations of our research and give a highlight of its forthcoming development.

We hope that this study will contribute to a better understanding of proof learning in higher education, more specifically, on the influence of student conceptions on this object and on the polysemy of the words attached to it, in the evolution of their ability to prove mathematical results.

1) Theorical framework

In mathematical practices, some polysemic words are used, for example hypothesis, proof, justification. Some terms are used in French but not in other languages. It is the case of the French word *Démonstration* that has no English translation. Texts, written by the French Ministry of National Education which guide mathematical teaching in secondary level use and define the word *Démontrer* as "use a deductive reasoning and some rules […] to reach some conclusion" (Ministère de l'éducation nationale d. l., 2015). However, in mathematical education research, many terms are used to talk about reasoning and proving: explanation, justification, proof, validation, for example. In their practices, mathematics teachers use many terms when mathematical reasoning is concerned (Baron & Hache, 2019). The difference between these words does not explain and highlights the different conceptions of teachers about justification, explanation or proof. At the secondary level, conceptions of students about these concepts are mainly built on conceptions of their teachers. When asking to justify, students refer to many expectations of different teachers about the structure of the answers or the process to reach a conclusion.

The question of the meaning of some words around the concept of proof is widely discussed in research in mathematics education. Cabassut (2005) highlights the various meanings of some words in different languages, French, English and German. Balacheff give a definition about the polysemic words "explanation" "proof" or "reasoning" (Balacheff, 1987). Due to the absence of translation of *Démontrer*, Balacheff talks about a "mathematical proof" (Balacheff, 1988). Regarding the word justification, this same author considers two types of explanation, explanation to convince and explanation to prove (Balacheff & Soury-Lavergne, 1996). Due to this polysemy, we are interested in the link between multilingualism context and proof learning.

As our study is conducted in French ultramarine territories, we point out some cultural and linguistic specificities of these territories. Lavigne (2011) highlights in New Caledonia the impact of interculturality for the learning of mathematics. In the case of the French West Indies, Arneton, Bocéréan, & Flieller (2013) show that the impact of bilingualism on mathematical learning is nonsignificant at the beginning of the secondary level. However, according to the curricula, the learning of proof begins in the middle of the secondary level: The students considered in this study have not learned yet to prove. Thus, these results are not contradictory with those of Cabassut (2005), Hache (2019) and Vergnaud (1991) which show that an intercultural context has an impact when considering the definition of some words in mathematical learning. In the same vein, the anthropological approach of Ogbu & Simons (1998) highlights the importance of meanings in a multicultural context. We can also quote the work of Borba (1990) about the impact of such as multicultural context on mathematical knowledge. Following these anthropological approaches, we are interested in the link between meanings of some terms and learning of proof at the transition between secondary schools and higher education in French West Indies.

Moreover, according to Selden & Selden (2003) about the difficulties of students at the transition to higher education, the expectations of university teachers about proof are closer to the conceptions of mathematicians. Thus, students face various expectations about proof and proving, some based on the conceptions of proof in the secondary level and some linked with the mathematician's practices. However, Weber (2001) highlights that students' difficulties at the higher education level is that they need to have proper conceptions about what is a proof. Due to the importance of the meaning of concepts for a better understanding of the expectations, we are interested in their conceptions of the meaning of some polysemic words.

2) Research Methodology

Following the anthropological framework of Chevallard (1985) about the didactic transposition, we consider the importance of the words used in the teaching text. In the French mathematical education, when proof learning is studied, the polysemy of some terms, not used in the English context, may become a difficulty. It's why we are interested by the meanings of words used in the teaching text about mathematical proof and the conceptions of students on these meanings and concepts. The proof expected by teachers at the higher education level is close to mathematician expert practices and are different from the academic expectations which guided the learning of proof at secondary level. To study the learning of proof in higher education, we have to consider this difference, the difficulties of students in the transition to higher education level and the gap between secondary level and higher education about teaching practices and expectations about proving a result. More specifically, we focus on the link between meanings that students give to the French words Démonstration and Justification and on the evolution of their ability to proof a result.

We assume that students have various conceptions of the meaning of these terms that evolve during the first years of higher education. We also put forward the hypothesis that adequacy can be noted between conceptions of the meaning of these terms and perception of the evolution of their ability to prove a result in mathematics.

Our research context is the higher education in the French West Indies. In these ultramarine territories, teaching is guided by national instructions of the French Ministry of National Education. The culture of these French departments leads to some linguistic specificities that could influence the conceptions of students about the meanings of some words involved in their learning.

To study the evolution of learning proof and meaning of some terms we have conducted a longitudinal survey. The students surveyed follow a course in the French West Indies in which mathematics occupies a significant hourly share. They complete a degree in Mathematics at the French West Indies University or follow a course preparing them to entry into engineering school after two or three years. We ask them to fill a questionnaire about the meaning of the terms *Démonstration* and *Justification* at three different times: at the beginning of the first year (time T0), at the beginning of the second year (time T1) and at the end of the second year (time T2). Thus, we can observe the evolution of their conceptions. At these same three times, we ask them to rate their perceived difficulties when they have to find arguments in mathematics and write a proof, using a Lickert scale. In the last step (time T2), they must also rate the evolution of their ability to prove.

With these results, we observe the evolution of their ability to prove a result and their difficulties in arguing, reasoning or write a proof, studying the difference between the means of responses to the questionnaire at the beginning of the first and the second year. We use a Student t-test to check the statistical significance of the results (Escofier & Pagès, 2008).

We also establish a causal link between these perceptions and the evolution of conceptions about the meaning of the two terms, conducted with an implicative statistical analysis (Gras & Regnier, 2017). For example, this allows us to write some implications such as "if the subject think he is able to write a proof at the first step of the survey, then he thinks the meaning of *Démontrer* is close to a proof". These kinds of implications are established with a certain probability of error, described with a number named the implication intensity. The results are given in terms of quasi-implication, represented by an implicative graph.

Giornale Italiano di Educazione alla Salute, Sport e Didattica Inclusiva / Italian Journal of Health Education, Sports and Inclusive Didactics - Anno 6 n. 4 - ISSN 2532-3296 ISBN 978-88-6022-456-9 - ottobre - dicembre 2022 - CC BY-NC-ND 3.0 IT- https://doi.org/10.32043/gsd.v6i4.706

3) Results of research

a) The meaning of the words Démonstration and Justification

We have questioned 168 students who began their course in higher education in 2019 and 2020. One hundred and thirty-eight of them give an answer in the first year. Forty-four students of these groups (2019 and 2020) have answered the questionnaire at the three steps of the study. The loss of respondents during the process is principally due to students who change courses at the end of the first year and, probably, to the effects of the Covid crisis.

In this study, we choose to treat together the two promotions of students to minimizing the biasing effect due to teaching practices. It's why we consider a study population consisted of 44 students, 18 of these students complete a degree in Mathematics and 26 follow courses preparing to entry into engineering schools.

Regarding the word *Démonstration*, we note six different meanings given by the respondents (Table 1): "Calculation", "Reasoning", "Validation", "Proof", "Process" and "Explanation".

The group of meaning "Validation" contains answers that refer to say if a mathematical statement is true or false or to test a hypothesis. The one titled "Process" contains answers that refer to a deductive process in two steps with a formal structure. We note that the distribution of these conceptions varies among the steps of the study.

Démonstration	beginning of the course	beginning of the second year	end of the second year
Calculation	2	0	1
Reasoning	8	7	6
Validation	10	8	7
Proof	12	16	9
Process	1	6	10
Explanation	8	5	3

Table 1: distributions of meanings about *Démonstration* at the three steps of the study

Regarding the word *Justification*, we note four different meanings given by the respondents (Table 2): "Argumentation", "Proof", "Explanation" and "Validation". The group of meaning titled "Argumentation" contains answers that refer to find arguments. The one titled "Validation" contains answers about testing a hypothesis or saying if a statement is true or false. The evolution of these conceptions concerns principally the meaning of this word close to validation and explanation.

Justification	beginning of the course	beginning of the second year	end of the second year
Argumentation	2	0	1
Proof	8	7	6
Explanation	10	8	7
Validation	12	16	9

Table 2: distribution of meanings about *Justification* at the three steps of the study

We note that some meanings about *Démonstration* are given for meaning about *Justification.* We can think that the meaning of the word *Justification* is close to the meaning of the word *Démonstration* for some students.

b) Difference of means in answers at the beginning of the first and second years

We note that the conceptions of students about the meanings of the words *Démonstration* and *Justification* evolve in the first years of their course. Even if the concept of proof is not taught in these courses, the evolution of these conceptions leads us to think that this learning continues at the higher education level. To observe this evolution, we study the difference between responses to the questionnaire at each step about their abilities in proving a result.

Between the beginning and the end of the first year, fewer students feel able to find arguments to prove a result (ability to find arguments) and get discouraged when faced with a difficulty in mathematical reasoning (Table 3). About writing a proof, fewer students think they have few difficulties to write a proof.

	T0	**T1**
ability to find arguments	23	18
reasoning discouragement	30	28
great difficulties in writing a proof	5	6
difficulties in writing a proof	14	29
few difficulties in writing a proof	22	8
no difficulties in writing a proof	0	0

Table 3: Evolution of ability about proving a result between T0 and T1

The answers of students about the evolution of their abilities between the beginning of the course and the end of the second-year show that the greater part of them have improved their abilities to write a proof and find arguments to prove a result (Table 4).

	same difficulties	**no difficulties as at the beginning**	**improving abilities**	**worsening abilities**
find arguments	4	8	28	4
write a proof	7	3	28	4

Table 4: Perception of students about the evolution of their abilities

With a Student t-test, we check if the means of the answers about finding arguments, reasoning, and writing a proof is different at the beginning of the first and at the beginning of the second year. Studying 44 answers, we compare the Student t-value for a 95% confidence interval with degrees of freedom equal to 43. With the Student distribution table, the observed value,

calculate using SPSS, must be greater than 2,017 for concluding to a difference between the means at the beginning of the first year and at the beginning of the second year.

About finding arguments, we note that the t-value observed is equal to 1,301 (Table 5). Moreover, the p-value involves that the chance to have a difference meaning equal to 0,11364 and equal means between T0 and T1 is equal to 20 in 100 (Table 5). It is why we cannot reject the hypothesis that we have the same mean at T0 and at T1.

	Difference of mean	95% Confidence Interval of the Difference		**t**	Degrees of freedom	p-value
		Lower	Upper			
argumentsT0 - argumentsT1	,11364	-,06250	,28977	**1,301**	43	,200

Table 5: Student's t test about finding arguments at T0 and T1

About reasoning, we study the ability not to be discouraged when they have a difficulty in mathematical reasoning. We note that the t-value observed is equal to 0,573 (Table 6). Moreover, the p-value involves that the chance to have a difference meaning equal to 0,4545 and equal means between T0 and T1 is equal to 57 in 100 (Table 6). It is why we cannot reject the hypothesis that we have the same mean at T0 and at T1.

	Difference of mean	95% Confidence Interval of the Difference		**t**	Degrees of freedom	p-value
		Lower	Upper			
decraisonnementT0 - decraisonnementT1	,04545	-,11454	,20545	**,573**	43	,570

Table 6: Student's test about mathematical reasoning

About writing a proof, we note that the t-value observed is equal to 1,45 (Table 7). Moreover, the p-value involves that the chance to have a difference meaning equal to 0,25 and equal means between T0 and T1 is equal to 15,4 in 100 (Table 7). It is why we cannot reject the hypothesis that we have the same mean at T0 and at T1.

	Difference of mean	95% Confidence Interval of the Difference		**t**	Degrees of freedom	p-value
		Lower	Upper			
redactionT0 - redactionT1	,25000	-,09773	,59773	**1,450**	43	,154

Table 7: Student's test about writing a proof

c) Causal links between perception of abilities and conceptions about the meaning of words

The Student t-test shows that we cannot find a significative difference between the observed means in the score of the answers of the students about their difficulties in proving a result. However, the evolution of conceptions of students show evolution in this learning. To describe this evolution, using an implicative statistical analysis, we look for a causal link between the perceptions of students about their difficulties about proving and their meaning about the two

words *Démonstration* and *Justification*. We choose to look for quasi-implications with a confidence level greater than 85%.

The first causal link we can note is about the evolution of ability to write a proof and to argue. Students who have improved their ability to write a proof between the beginning of their course and the end of the second year in higher education have also improved their ability to find arguments to prove a result (Figure 1).

Regarding the meaning of the word *Démonstration*, we note a link between students who give to the word *Démonstration* meaning close to "reasoning" at the end of the second year have improved their ability to write a proof (Figure 1).

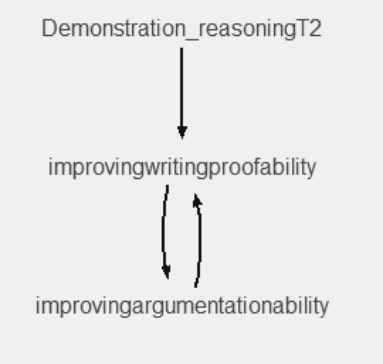

Figure 1: Causal links between meanings about *Démonstration* and evolution of ability in writing proof

We also note that students who give to the word *Démonstration* meaning close to "proof" at the beginning of their course and at the end of the second year do not be discouraged when they have a difficulty in mathematical reasoning at the end of the second year (Figure 2).

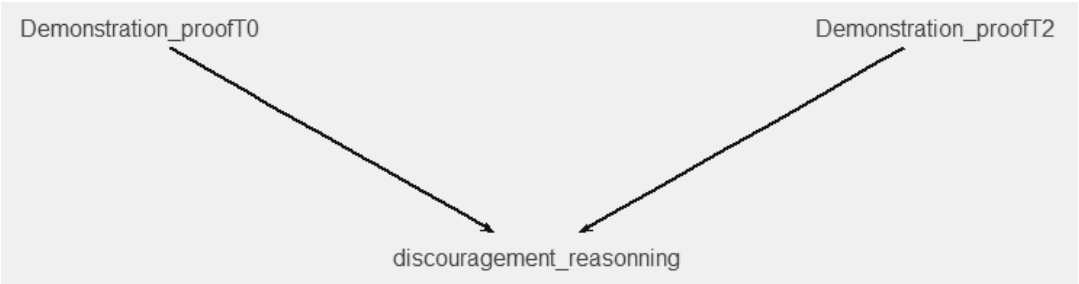

Figure 2: Causal links between meanings about *Démonstration* and evolution of ability in reasoning

Regarding the word *Justification*, we can note that students who give to *Justification* meaning close to find arguments at the beginning of their course and students who give to this word a meaning close to proof at the end of the second year have improved their ability to write a proof (Figure 3).

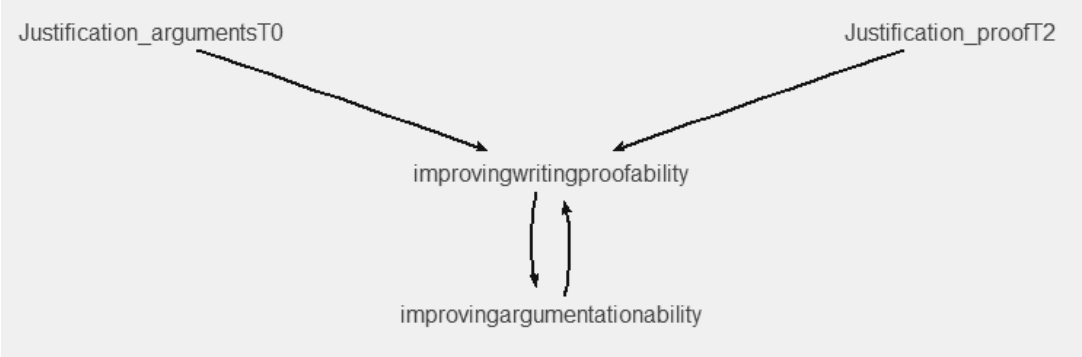

Figure 3: Causal links between meanings about *Justification* and evolution of ability in writing proof

We also note that students who give to this word a meaning close to "explanation" at the beginning of the second year have improved their ability to find arguments to prove a result (Figure 4).

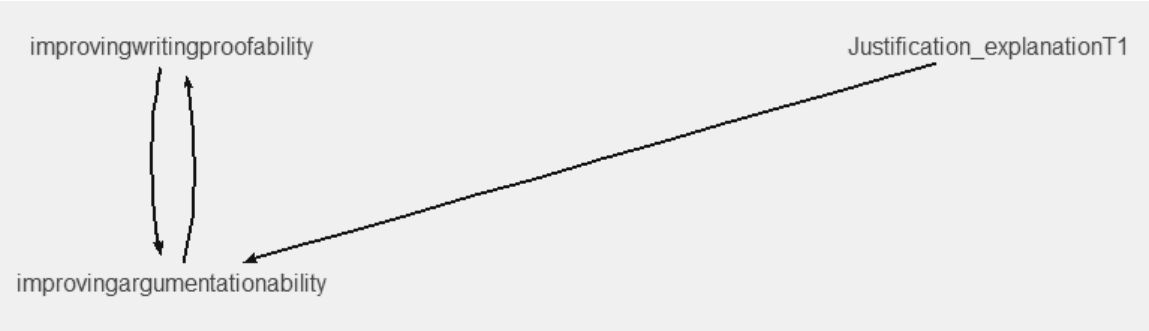

Figure 4: Causal links between meanings about *Justification* and evolution of ability in arguing

With these results, we will be able to describe the evolution of the learning of proof and the influence of conceptions of students about the meaning of these two words on this learning.

4) Discussion of results

We recall that the aim of this study is to the following hypotheses: First, we assume that the students have various conceptions of the meaning of these terms that evolve during the first years of higher education; Secondly, we assume that we can note adequacy between conceptions of the meaning of these terms and perception of the evolution of their ability.

In their answers, students give six groups of meaning about the word *Démonstration*. Studying these answers, we can note a difference of conception at the beginning and after two years. At the beginning of their course in higher education, the responding students think that the meaning of *Démonstration* is close to a validation or a proof. At the beginning of the second year, most of the responding students think that *Démonstration* is close to a proof. At the end of the two years, more of them think that *Démonstration* is close to a process. We can think that, during these two years, the conception of students about the meaning of this word evolves from checking if a statement if true or false or testing a hypothesis to a process that contains steps that have a formal structure.

Regarding the word *Justification*, students give four groups of meaning. In these answers, we can note evolution in the conception of the respondents. At the beginning of their course, *Justification* has a meaning close to an explanation or "Validation". At the beginning and the end of the second year, they that the meaning of *Justification* is close to "Validation". We can also note an important part of students who have not responded to this question at the end of the second year. We can think that, during the first year, the conception of students about the meaning of the word *Justification* evolves from an explanation to check if a statement is true or false or test a hypothesis.

It is the reason why we can say that, in accordance with our first hypothesis, the meaning of the words *Démonstration* and *Justification* evolve between the beginning of the course and the end of the second year in higher education. We can also note the polysemy of these words for the students.

The answers of students show the evolution of their abilities to find arguments to prove a result and write a proof between the beginning of the first and the second year. Moreover, when they rate their evolution between the beginning of their course and the end of the second year, the greater part of them feels they have improved their ability to find arguments to prove a result and write a proof. We can think that the learning of proof evolves during the first and the second year as higher education students. However, studying the means of answers about their abilities in proving a result, the Student t-test does not reveal a significative difference between the conceptions of students at the beginning of their course and at the beginning of the second year.

We have therefore looked for causal links between their perceptions about the meaning of the words *Démonstration* and *Justification* and the evolution of their ability to prove a result. An implicative statistical analysis shows us that, according to our second hypothesis, we can find an adequation between conceptions of students about the meaning of some polysemic words and their perception of the evolution of their ability to prove a result.

Conclusions

The curricula, which guides the learning of proof at the secondary level in France, does not use the term "proof", but the French word *Démonstration*. However, many polysemic words are used in teaching practices with respect to that matter. At higher education level, the learning of how to prove a result is less mentioned in official instructions about mathematical teaching. Conceptions of students at this step of their course is mainly built on conceptions of the secondary level teachers. But our results show that this learning does not stop at the end of the secondary level. First, students have various conceptions about the meaning of the French words *Démonstration* and *Justification*. Second, their conception about these meanings changes between the beginning of their course and the end of the second year in higher education. Their perceptions about their ability to find arguments to prove a result and write a prove also evolves during these two steps. We can also find a causal link between the conception of students about the meaning of these words and the evolution of their abilities.

This study allowed us to highlight adequation between conceptions about polysemic words and evolution of abilities in the case of learning of proof in the context of French West Indies higher education. The obtained results suggest that, in this multicultural context, cultural factors have been shaping conceptions of students about the meaning of some words around proof.

We must note one limitation of our research. The small number of responses to the three steps of the study influences the result of the Student t-test and does not allow us to note a significant

difference between means of responses at the beginning of the course and the beginning of the second year. However, we made the choice consider together the answers of students who started their course in 2019 and 2020. This choice allows us to limit the biases related to the education of students at secondary level.

We hope this study will lead to a better understanding of links between the polysemy of words, the conceptions of students about the meaning of these words and the learning of mathematical objects like proof. We want to extend this study to other ultramarine territories and observe links between some social criteria on this adequation.

References

Arneton, M., Bocéréan, C., & Flieller, A. (2013, March). Les performances en mathématiques des élèves des départements d'Outre-mer . L'Orientation scolaire et professionnelle. doi:10.4000/osp.4029

Balacheff, N. (1987). Processus de preuve et situations de validation (Proving Processes and Situations for Validation). Educational Studies in Mathematics, 18, 147–176.

Balacheff, N. (1988). A study of students' proving processes at the junior high school level. Second UCSMP international conference on mathematics education. Chicago: NCTM.

Balacheff, N., & Soury-Lavergne, S. (1996). Explication et préceptorat, à propos d'une étude de cas dans TéléCabri. Explication et EIAO, actes de la journée du 26 janvier 1996 (PRC-IA) (pp. 37-50). Paris: Université Blaise Pascal.

Baron, E., & Hache, C. (2019). Expliquer, justifier, prouver, Démontrer? Repères IREM.

Borba, M. C. (1990). Ethnomathematics and education. For the learning of mathematics, 10, 39–43.

Cabassut, R. (2005, May). Proof, reasoning and validation in mathematical teaching of French and German secondary schools. Theses, Université Paris-Diderot - Paris VII.

Chevallard, Y. (1985). La transposition didactique : du savoir savant au savoir enseigné. Grenoble: Éd. la Pensée sauvage.

Chevallard, Y. (1991). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique.

Escofier, B., & Pagès, J. (2008). Analyses factorielles simples et multiples. Objectifs méthodes et interprétation. Dunod.

Gras, R., & Regnier, J.-C. (2017). Analyse implicative des variables binaires. Intensité implicative. Intensité entropique (Partie 1, Ch. 1). Dans R. Gras, J.-C. Régnier, F. Guillet, D. Lahanier-Reuter, & C. Marinica (Éds.), L'analyse statistique implicative: Des Sciences dures aux Sciences Humaines et Sociales. Cépaduès Éditions.

Hache, C. (2019, December). Questions langagières dans l'enseignement et l'apprentissage des mathématiques. Habilitation à diriger des recherches, Université Paris Diderot.

Lavigne, G. (2011). Langues et mathématiques à l'école dans les cultures océaniennes. doi:10.6098/2012NCAL0041

Ministère de l'éducation nationale, d. l. (2015, November). Bulletin officiel spécial. Le Bulletin Officiel de l'Education Nationale.

Ogbu, J. U., & Simons, H. D. (1998). Voluntary and Involuntary Minorities: A Cultural-Ecological Theory of School Performance with Some Implications for Education. Anthropology & Education Quarterly, 29, 155-188.

Selden, A., & Selden, J. (2003, January). Validations of Proofs Considered as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem? Journal for Research in Mathematics Education, 34, 4. doi:10.2307/30034698

Vergnaud, G. (1991). Langage et pensée dans l'apprentissage des mathématiques. Revue française de pédagogie, 96, 79–86. doi:10.3406/rfp.1991.1350

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. Educational Studies in Mathematics, 48, 101–119. doi:10.1023/a:1015535614355